Anoka-Hennepin Secondary Curriculum Unit Plan

Department:	Mathematics	Course:	Intermediate Algebra	Unit 7 Title:	Root Functions and Radical Equations	Grade Level(s):	9
Assessed Trimester:	Trimester B	Pacing:	18-19 Days	Date Created:	6/13/2013	Last Revision Date:	08/27/2014

Course Understandings: *Students will understand that:*

A. Relationships exist between real-world situations, mathematical equations, inequalities and graphs for linear, exponential, absolute value, radical, and polynomial functions.
C. There are a variety of strategies of varying efficiency for simplifying linear, absolute value, exponential, radical, complex and polynomial expressions.
E. The context of a problem is important in recognizing the reasonableness of a solution.
F. There are benefits and limitations in the use of calculators and other technology to solve mathematical situations

DESIRED RESULTS (Stage 1) - WHAT WE WANT STUDENT TO KNOW AND BE ABLE TO DO?

Established Goals

Minnesota State/Local/Technology Standard(s) addressed (2007):

- **Standard (9.2.1.#):** Understand the concept of function, and identify important features of functions and other relations using symbolic and graphical methods where appropriate.
  Benchmark:
  9.2.1.3 Find the domain of a function defined symbolically, graphically or in a real-world context.
  9.2.1.6 Identify intercepts, zeros, maxima, minima and intervals of increase and decrease from the graph of a function.

- **Standard (9.2.2.# - Modified):** Represent real-world situations and verbal descriptions involving functions of n^{th} roots-with tables, symbols and graphs.
  Benchmark:
  9.2.2.6 Sketch the graphs of common non-linear functions such as $f(x) = \sqrt{x}, f(x) = |x|$, $f(x) = \frac{1}{x}$, $f(x) = x^3$, and translations of these functions, such as $f(x) = \sqrt{x-2} + 4$. Know how to use graphing technology to graph these functions.

- **Standard (9.2.3.# - Modified):** Generate equivalent algebraic expressions involving radicals; use algebraic properties to evaluate expressions.
  Benchmark:
  9.2.3.1 Evaluate polynomial and rational expressions and expressions containing radicals and absolute values at specified points in their domains.
  9.2.3.6 Apply the properties of positive and negative rational exponents to generate equivalent algebraic expressions, including those involving n^{th} roots.
  9.2.3.7 Justify steps in generating equivalent expressions by identifying the properties used. Use substitution to check the equality of expressions for some particular values of the variables; recognize that checking with substitution does not guarantee equality of expressions for all values of the variables.

- **Standard (9.2.4.# - Modified):** Solve n^{th} root equations symbolically and graphically. Interpret solutions in the original context.
  Benchmark:
  9.2.4.7 Solve equations that contain radical expressions. Recognize that extraneous solutions may arise when using symbolic methods.
  9.2.4.8 Assess the reasonableness of a solution in its given context and compare the solution to appropriate graphical or numerical estimates; interpret a solution in the original context.

Transfer

Students will be able to independently use their learning to: (product, high order reasoning)

- Model, analyze and solve real world situations using square root functions.

Meaning

Unit Understanding(s):	Essential Question(s):
Students will understand that: ● A real-world situation can be represented as a n^{th} root function and will demonstrate an understanding of how to find reasonable solutions.	**Students will keep considering:** ● Where can I find situations involving n^{th} root functions in the real world? ● When looking at square root functions, how do the significant features of the graph, its algebraic equation and real world representation relate to each other? ● How do I decide the best method to solve a n^{th} root equation? ● How do the skills and knowledge that we are learning influence the task of understanding situations that can be modeled by n^{th} root functions?

Acquisition

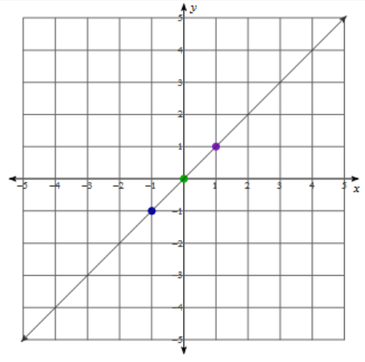
Knowledge - Students will:
- Demonstrate understanding of radical expressions and expressions with rational exponents.
- Demonstrate understanding of the significant features of its graph (intercepts, domain and range, intervals of increase and decrease)

Reasoning - Students will:
- Understand the meaning of the domain and range of a function symbolically, graphically or in a real-world context.
- Justify steps in generating equivalent expressions by identifying the properties used.
- Use substitution to check the equality of expressions for some particular values of the variables and recognize that checking with substitution does not guarantee equality of expressions for all values of the variables. (Recognize that extraneous solutions may arise when using symbolic methods.)
- Assess the reasonableness of a solution in its given context and compare the solution to appropriate graphical or numerical estimates; interpret a solution in the original context.

Skills - Students will:
- Graph square root and cube root functions
- Solve equations with radical expressions and expressions with rational exponents.
- Evaluate radical expressions at specified points in their domains.
- Apply the properties of positive, negative, and rational exponents to generate equivalent algebraic expressions, including those involving n^{th} roots.
  - *For example:* $\sqrt{2} \times \sqrt{7} = 2^{\frac{1}{2}} \times 7^{\frac{1}{2}} = 14^{\frac{1}{2}} = \sqrt{14}$. Rules for computing directly with radicals may also be used: $\sqrt[3]{2} \times \sqrt[3]{x} = \sqrt[3]{2x}$.
- Solve equations that contain radical expressions algebraically and graphically
  - *For example*: The equation $\sqrt{x-9} = 9\sqrt{x}$ may be solved by squaring both sides to obtain $x - 9 = 81x$, which has the solution $x = -\frac{9}{80}$. However, this is not a solution of the original equation, so it is an extraneous solution that should be discarded. The original equation has no solution in this case.
  - *Another example*: Solve $\sqrt{3-x} + 1 = -5$.

Common Misunderstandings
- Students do not know how to find a decimal approximation for radical expressions using their calculators.
- Students will square expressions involving radicals incorrectly. *For example,* $(3\sqrt{x})^2 = 3x$ *or* $(\sqrt{m}+5)^2 = m + 25$.
- Students may not take the real-world context for a relationship into account when giving solutions.
- Students will incorrectly add radical expressions with unlike radicands (i.e $\sqrt{3} + \sqrt{2} = \sqrt{5}$).
- Students sometimes do not recognize implied grouping symbols when evaluating expressions involving radicals and fractions. A student who is evaluating the expression $x - \frac{3}{2}x + 7 \; for \; x = 5$ might enter in his calculator $5 - 3/2 * 5 + 7$ and get an incorrect answer of 4 .5 rather than entering $\frac{(5-3)}{(2*5+7)}$ and getting the correct answer of 217. Some students incorrectly evaluate the expression $\sqrt{2x+3}$ for $x = 8$ as $\sqrt{2*8+3}$ and get the incorrect result $\sqrt{19}$ rather than the correct answer of 7.
- Students will calculate too small a set of point to graph a function. For example, a student asked to graph $y = \sqrt[3]{x}$ will correctly calculate the ordered pairs (-1, -1), (0, 0) and (1, 1) and incorrectly graph a line passing through the points.

Essential new vocabulary
- Extraneous
- Irrational numbers
- n^{th} power
- n^{th} root
- Radical
- Undefined



- Students do not use the correct syntax when entering a function into a graphing utility. Students who are trying to graph $y = \frac{1}{x-3}$ often enter $1 \div x - 3$, which results in a graph of a different function $y = \frac{1}{x} - 3$.
- Students interchange the *x*-intercept and the *y*-intercept.
- Students state that *x* and *y*-intercepts are values rather than the coordinates of points on a graph. The *y*-intercept of the line *y = 2x + 7* is the coordinate (0,7) not the value 7. The x-intercepts of the function y = *3(x + 5)(2 - x)* are (-5,0) and (2,0) not -5 and 2. This is an important distinction since intercepts highlight important features of the relationship between two variables and not simply the value of one variable.
- Students use end points of a function within a specific graphing window to represent the maximum or minimum values of the function.
- Students confuse the meaning of exponents and incorrectly calculate the value of exponential expressions (e.g., $2^{-3} = -8$, $2^{0} = 0$, *or* $9^{\frac{1}{2}} = 4.5$).
- Restricting the domain of functions to avoid complex numbers is difficult for many students. Students do not necessarily have trouble finding the restricted values but fail to understand why square root functions are not defined for all values.